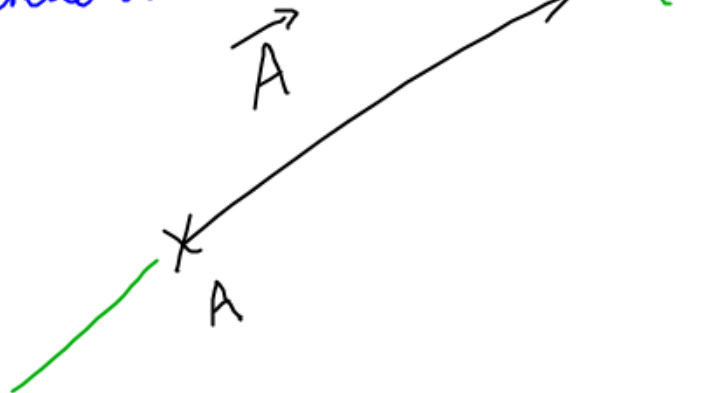


2. Vecteurs

Graphique

- point d'appl. A
- direction (support) (d)
- sens 
- norme (intensité) longueur

du vecteur dans une
échelle donnée (ex: $1\text{cm} \rightarrow 1\text{N}$) (d)



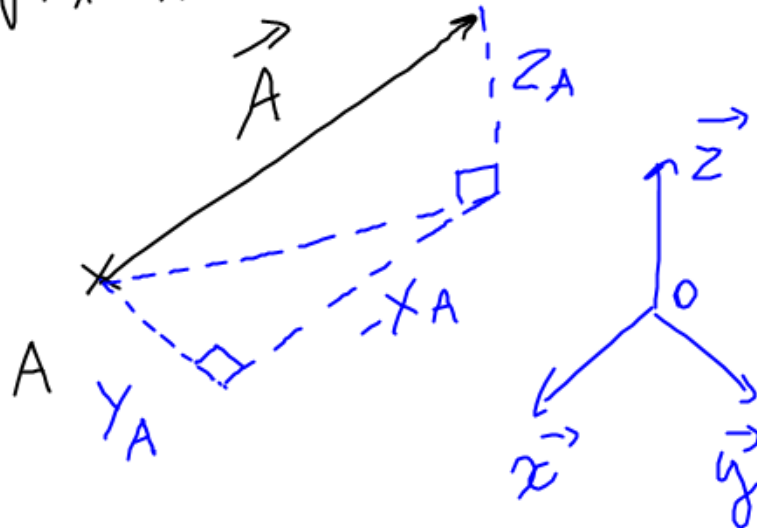
Analytique

- point d'application

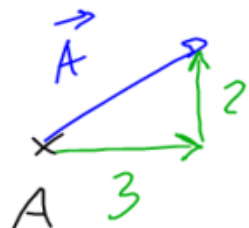
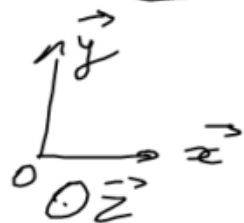
- coordonnées

$$\vec{A} = \begin{pmatrix} -X_A \\ Y_A \\ Z_A \end{pmatrix}$$

$$\|\vec{A}\| = \sqrt{X_A^2 + Y_A^2 + Z_A^2}$$



examples:



$$\vec{A} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$$

$$\|\vec{A}\| = \sqrt{3^2 + 2^2} \\ = \sqrt{13}$$

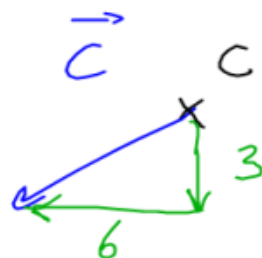
$$\vec{A} = 3 \cdot \vec{x} + 2 \cdot \vec{y}$$



$$\vec{B} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}$$

$$\|\vec{B}\| = \sqrt{4^2 + 4^2} \\ = \sqrt{32}$$

$$\vec{B} = 4 \cdot \vec{x} - 4 \cdot \vec{y}$$



$$\vec{C} = \begin{pmatrix} -6 \\ -3 \\ 0 \end{pmatrix}$$

$$\|\vec{C}\| = \sqrt{6^2 + 3^2} \\ = \sqrt{45}$$

$$\vec{C} = -6 \cdot \vec{x} - 3 \cdot \vec{y}$$

avec:

$$\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

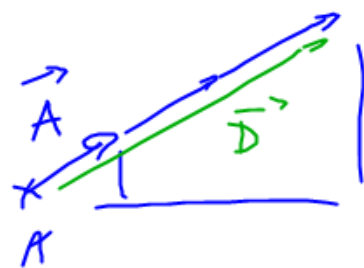
$$\vec{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\|\vec{x}\| = \|\vec{y}\| = \|\vec{z}\| = 1$$

$$\vec{D} = 3 \cdot \vec{A} = 3 \cdot \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$$

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Produkt skalare:

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos(\vec{u}, \vec{v})$$

$$\cos(\vec{u}, \vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$\vec{u} \cdot \vec{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = u_x v_x + u_y v_y + u_z v_z$$

$$\vec{A} \cdot \vec{x} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 3$$

$$\vec{A} \cdot \vec{y} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2$$

Produit scalaire:

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos(\vec{u}, \vec{v})$$

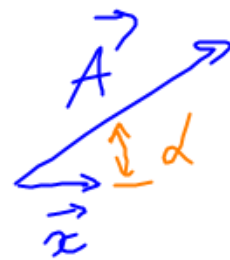
$$\cos(\vec{u}, \vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$\vec{u} \cdot \vec{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = u_x v_x + u_y v_y + u_z v_z$$

$$\vec{A} \cdot \vec{x} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 3$$

$$\vec{A} \cdot \vec{y} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2$$

Angle entre $\vec{A}$ et $\vec{x}$



$$\cos(\vec{A}, \vec{x}) = \frac{\vec{A} \cdot \vec{x}}{\|\vec{A}\| \cdot \|\vec{x}\|}$$

$$\cos \alpha = \frac{3}{\sqrt{13} \cdot 1}$$

$$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) \approx 33,7^\circ$$

Produit vectoriel :

$$\vec{u} \wedge \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \sin(\vec{u}, \vec{v}) \cdot \vec{w}$$

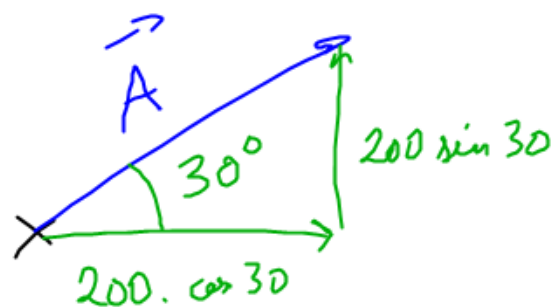
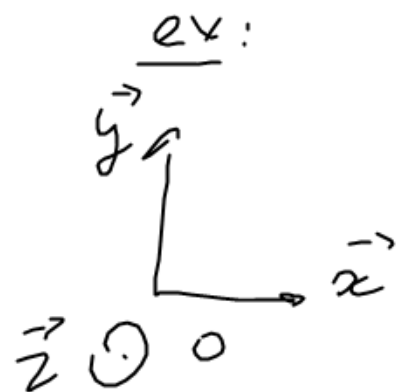
$\vec{w}$: vecteur unitaire
($\|\vec{w}\|=1$) normal
au plan formé par $\vec{u}$ et $\vec{v}$

$$\vec{u} \wedge \vec{v} = \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \end{pmatrix} = \begin{pmatrix} u_y \cdot v_z - u_z \cdot v_y \\ u_z \cdot v_x - u_x \cdot v_z \\ u_x \cdot v_y - u_y \cdot v_x \end{pmatrix}$$

(Note: In the original image, the first matrix is written with u_x, u_y, u_z in the first column and v_x, v_y, v_z in the second column. The second matrix shows the corresponding components of the cross product with signs: $u_y \cdot v_z - u_z \cdot v_y$, $u_z \cdot v_x - u_x \cdot v_z$, and $u_x \cdot v_y - u_y \cdot v_x$. There are also handwritten annotations: a green circle around the u_z term in the first row of the first matrix, a red circle around the $-$ sign in the third row of the first matrix, and a red circle around the $-$ sign in the third row of the second matrix.)

$$\text{ex: } \vec{x} \wedge \vec{y} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \vec{z} \quad \left| \quad \vec{y} \wedge \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = -\vec{z}$$

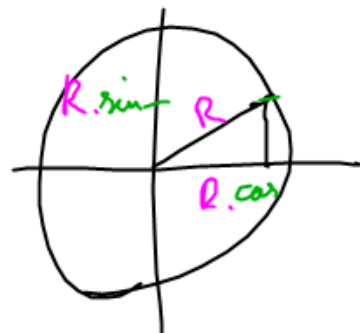




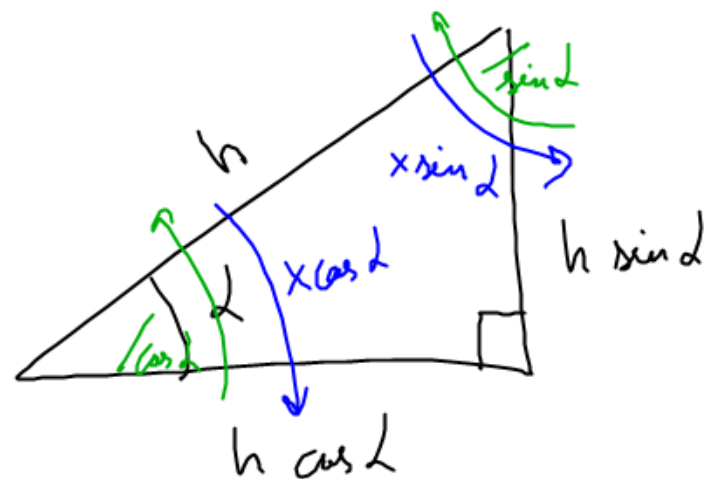
A

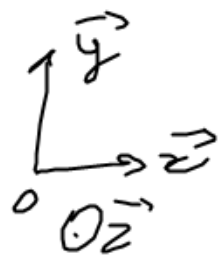
$$||\vec{A}|| = 200$$

$$\vec{A} = \begin{pmatrix} 200 \cos 30 \\ 200 \sin 30 \\ 0 \end{pmatrix}$$



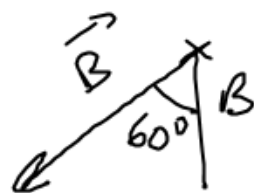
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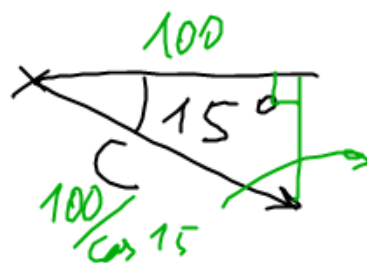
$$\|\vec{A}\| = 100$$

$$\vec{A} = \begin{pmatrix} \end{pmatrix}$$



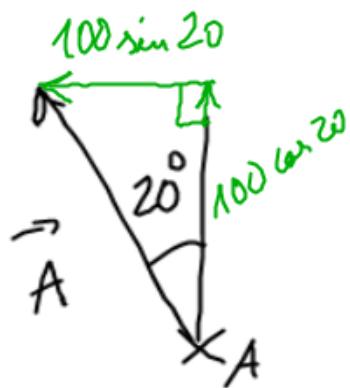
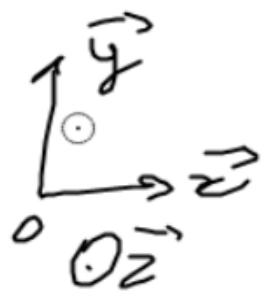
$$\|\vec{B}\| = 500$$

$$\vec{B} = \begin{pmatrix} \end{pmatrix}$$



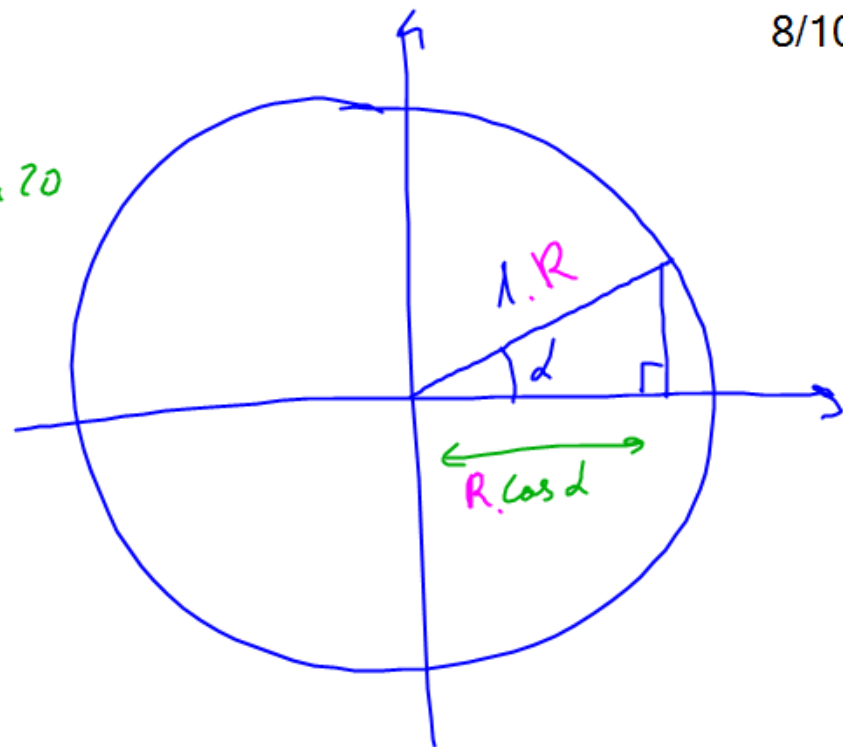
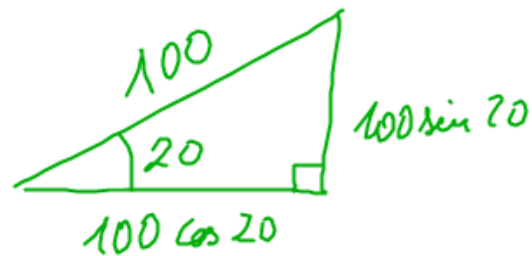
$$\vec{C} = \begin{pmatrix} 100 \\ -\frac{100 \sin 15}{\cos 15} \\ 0 \end{pmatrix}$$

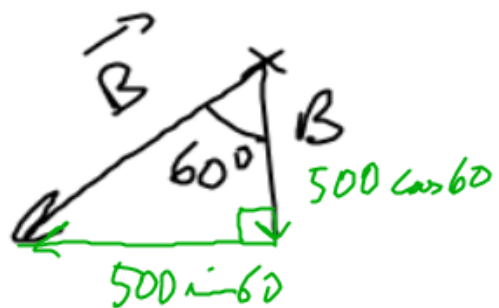
$$\|\vec{C}\| = \frac{100}{\cos 15}$$



$$\|\vec{A}\| = 100$$

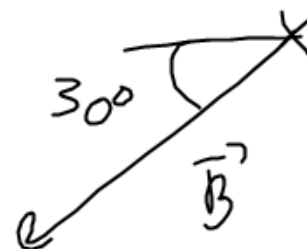
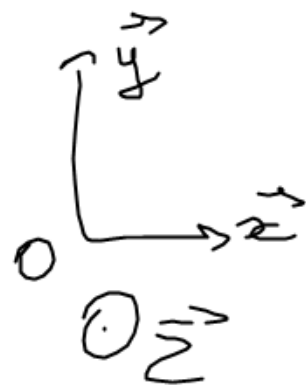
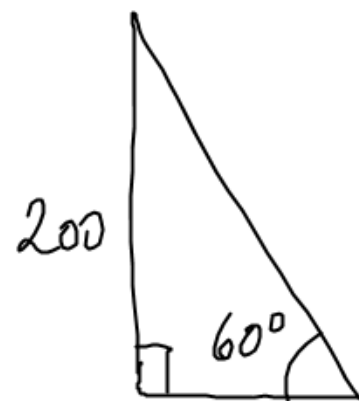
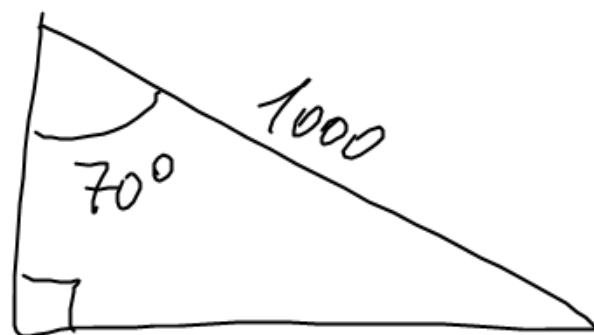
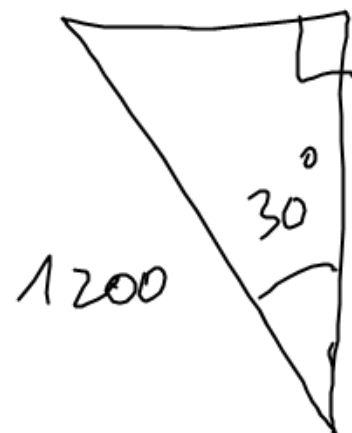
$$\vec{A} = \begin{pmatrix} -100 \sin 20 \\ 100 \cos 20 \\ 0 \end{pmatrix}$$





$$\|\vec{B}\| = 500$$

$$\vec{B} = \begin{pmatrix} -500 \sin 60 \\ -500 \cos 60 \\ 0 \end{pmatrix}$$



$$\|\vec{A}\| = 300$$

$$\vec{A} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\|\vec{B}\| = \dots$$

$$\vec{B} = \begin{pmatrix} -200 \\ \dots \\ \dots \end{pmatrix}$$