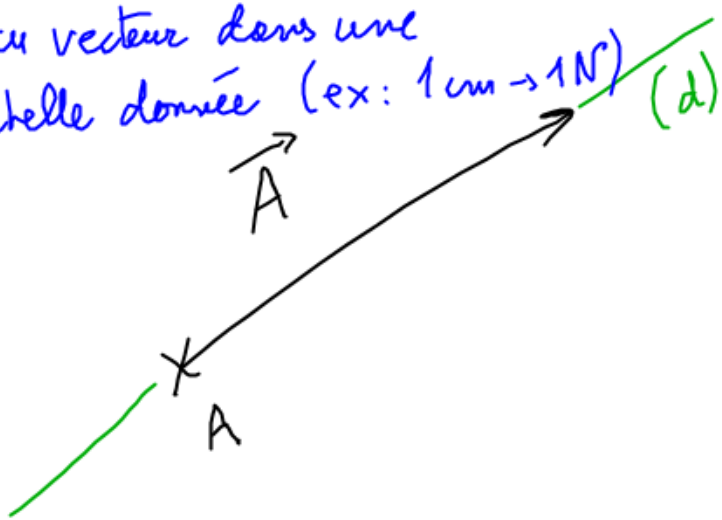
2. Vecteurs

Graphique

- point d'appl. A
- direction (support) (d)
- sens ↗
- norme (intensité) longueur du vecteur dans une échelle donnée (ex : 1 cm → 1 N)

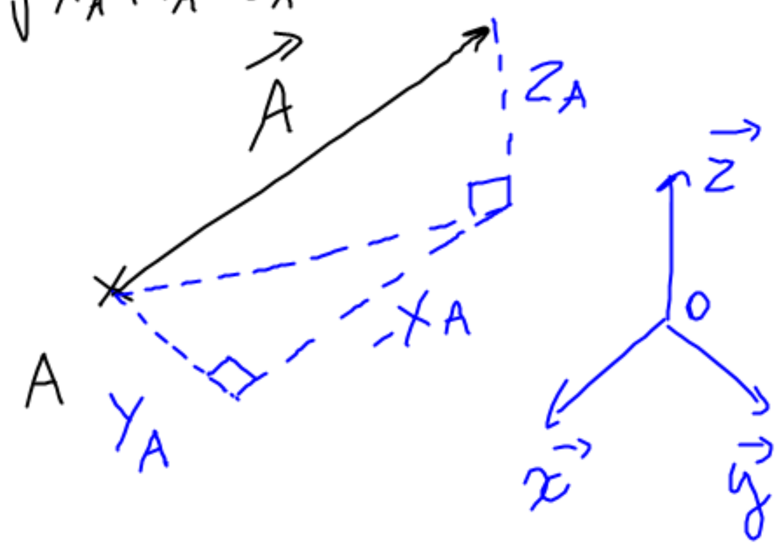


Analytique

- point d'application
- coordonnées

$$\vec{A} = \begin{pmatrix} -X_A \\ Y_A \\ Z_A \end{pmatrix}$$

$$\|\vec{A}\| = \sqrt{X_A^2 + Y_A^2 + Z_A^2}$$



exemples :

$$\vec{A} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix} \qquad \vec{C} = \begin{pmatrix} -6 \\ -3 \\ 0 \end{pmatrix}$$

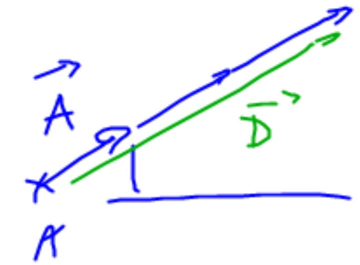
$$\|\vec{A}\| = \sqrt{3^2+2^2} = \sqrt{13} \qquad \|\vec{B}\| = \sqrt{4^2+4^2} = \sqrt{32} \qquad \|\vec{C}\| = \sqrt{6^2+3^2} = \sqrt{45}$$

$$\vec{A} = 3.\vec{x} + 2\vec{y} \qquad \vec{B} = 4\vec{x} - 4.\vec{y} \qquad \vec{C} = -6.\vec{x} - 3.\vec{y}$$

avec : $\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \vec{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\|\vec{x}\| = \|\vec{y}\| = \|\vec{z}\| = 1$$

$$\vec{D} = 3 . \vec{A} = 3 \cdot \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$$



Produit scalaire :

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos(\vec{u} \cdot \vec{v})$$

$$\cos(\vec{u}, \vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$\vec{u} \cdot \vec{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = u_x v_x + u_y v_y + u_z \cdot v_z$$

$$\vec{A} \cdot \vec{x} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 3$$

$$\vec{A} \cdot \vec{y} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2$$

Produit scalaire :

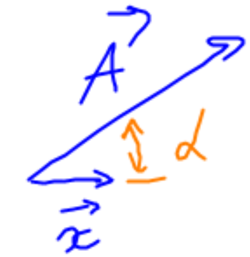
$$\vec{u}\cdot\vec{v} = \|\vec{u}\|.\|\vec{v}\|.\cos(\vec{u}\cdot\vec{v})$$

$$\vec{u}\cdot\vec{v} = \begin{pmatrix}u_x\\u_y\\u_z\end{pmatrix}\cdot\begin{pmatrix}v_x\\v_y\\v_z\end{pmatrix} = u_x v_x + u_y v_y + u_z . v_z$$

$$\cos(\vec{u},\vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|.\|\vec{v}\|}$$

$$\vec{A}\cdot\vec{x} = \begin{pmatrix}3\\2\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} = 3 \qquad \vec{A}\cdot\vec{y} = \begin{pmatrix}3\\2\\0\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} = 2$$

Angle entre $\vec{A}$ et $\vec{x}$



$$\cos(\vec{A},\vec{x}) = \frac{\vec{A}\cdot\vec{x}}{\|\vec{A}\|.\|\vec{x}\|}$$

$$\cos\alpha = \frac{3}{\sqrt{13}\,.\,1}$$

$$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) \simeq 33{,}7^{\circ}$$

Produit vectoriel :

$\vec{u} \wedge \vec{v} = \|\vec{u}\| . \|\vec{v}\| . \sin(\vec{u}, \vec{v}) . \vec{\omega}$

$\vec{\omega}$: vecteur unitaire ($\|\vec{\omega}\| = 1$) normal au plan formé par $\vec{u}$ et $\vec{v}$

$$\vec{u} \wedge \vec{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \wedge \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} u_y . v_z - u_z . v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y . v_x \end{pmatrix}$$

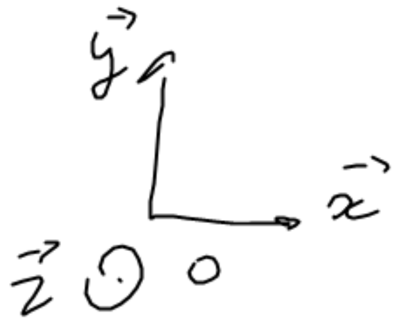
$u_x \quad - \quad v_x$

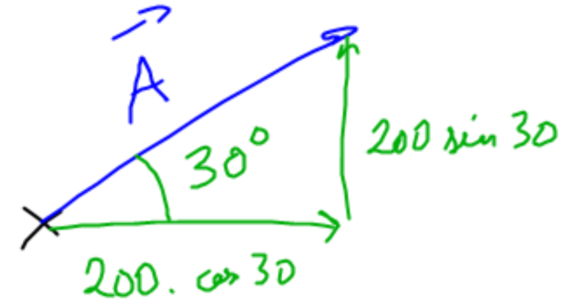
ex : $\vec{x} \wedge \vec{y} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \vec{z}$

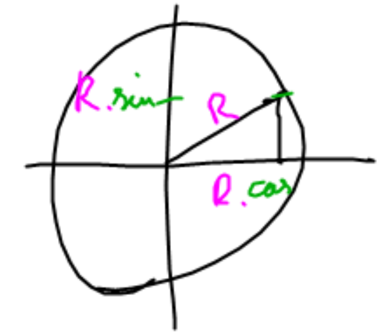
$1 \quad 0$

$\vec{y} \wedge \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = -\vec{z}$

ex:



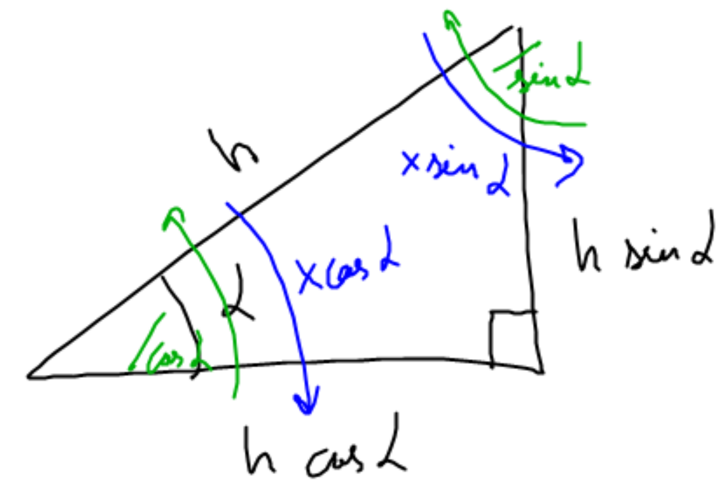


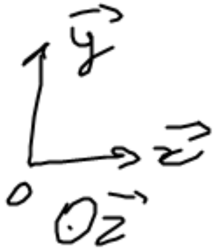


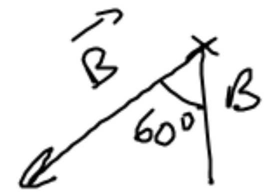
A

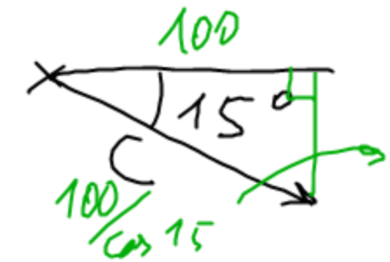
$\|\vec{A}\| = 200$

$$\vec{A} = \begin{pmatrix} 200\cos 30 \\ 200\sin 30 \\ 0 \end{pmatrix}$$









$$\|\vec{A}\| = 100$$

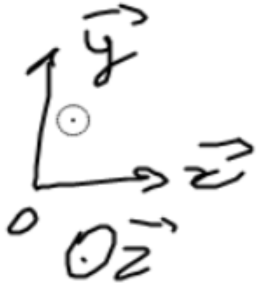
$$\vec{A} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

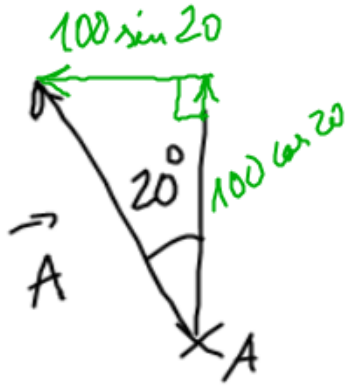
$$\|\vec{B}\| = 500$$

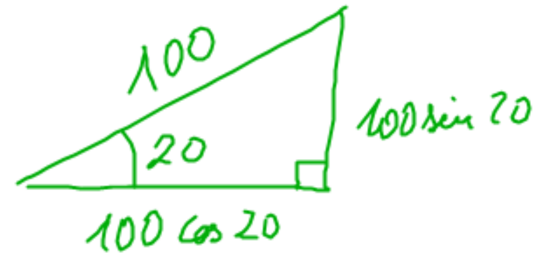
$$\vec{B} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

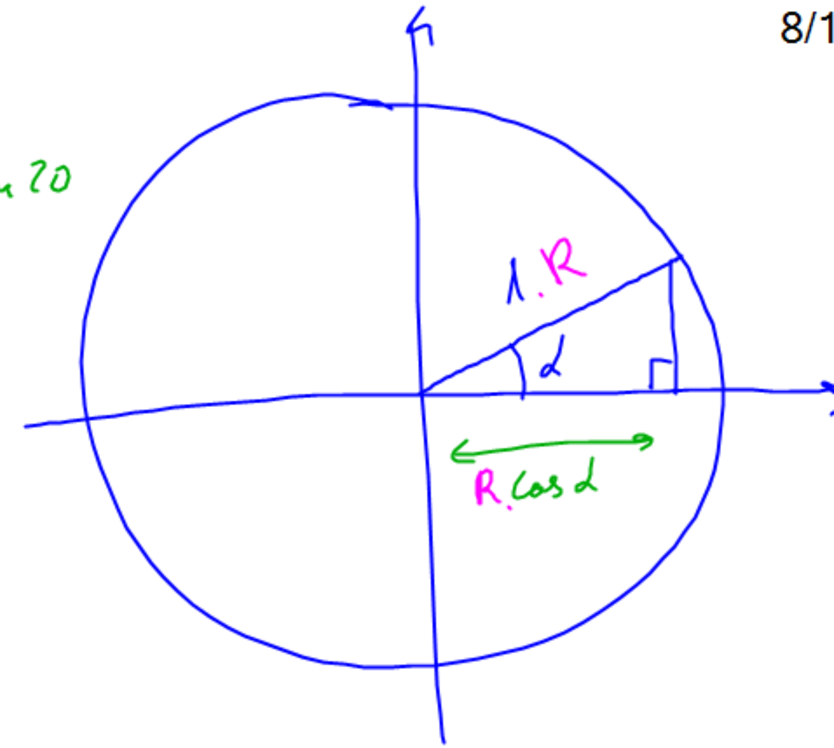
$$\vec{C} = \begin{pmatrix} 100 \\ -\dfrac{100 \sin 15}{\cos 15} \\ 0 \end{pmatrix}$$

$$\|\vec{C}\| = \frac{100}{\cos 15}$$



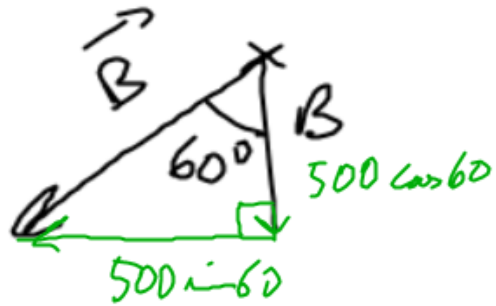






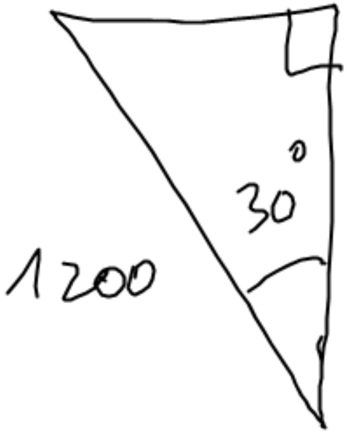
$$\|\vec{A}\| = 100$$

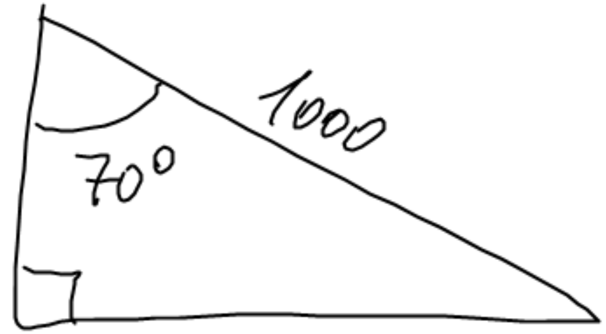
$$\vec{A} = \begin{pmatrix} -100 \sin 20 \\ 100 \cos 20 \\ 0 \end{pmatrix}$$

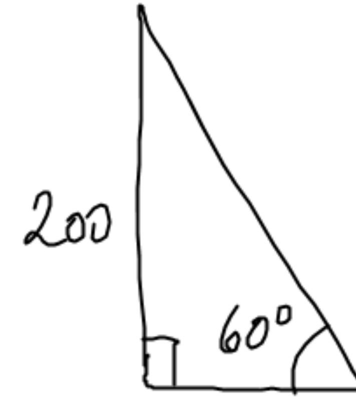


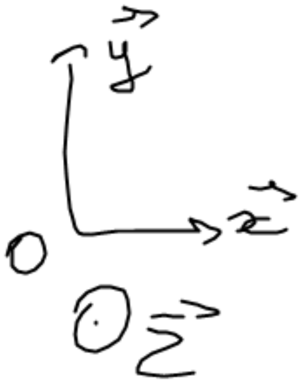
$$\|\vec{B}\| = 500$$

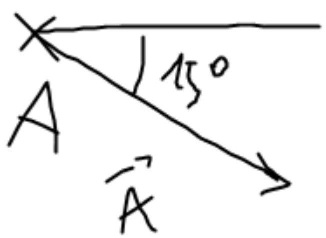
$$\vec{B} = \begin{pmatrix} -500\sin 60 \\ -500\cos 60 \\ 0 \end{pmatrix}$$

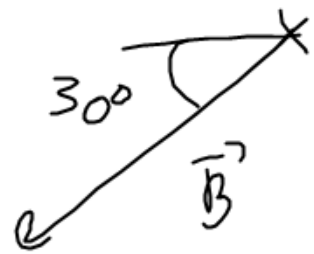












$\|\vec{A}\| = 300$

$\vec{A} = \begin{pmatrix} \\ \\ \end{pmatrix}$

$\|\vec{B}\| = \dots$

$\vec{B} = \begin{pmatrix} -200 \\ \dots \\ \dots \end{pmatrix}$